This formula is valid under the following conditions: the force of frontal air resistance to the motion of the projectile is proportional to the its velocity squared; wind speed components are much smaller than the horizontal component of projectile velocity; the projectile velocity projections on the Oy and Oz axes are much smaller than the projections on the Ox axis; the dimensionless coefficient of resistance and the magnitude of the crosswind are constant values.

However, in reality, the force of frontal air resistance to the motion of the projectile is only sometimes proportional to the its velocity squared; the projectile velocity projections on the Oz axis may be are much smaller than the projections on the Ox axis and may even be greater than it; the coefficient of resistance is depends on the value of the Makh number, so it can be considered constant only when shooting at short distances.

The authors propose a mathematical model for determining the magnitude of the lateral displacement of the projectile under the action of crosswinds. It is believed that the force of the crosswind on the projectile depends on the following factors: air density; the maximum area of the longitudinal section of the projectile; the difference between the value of the lateral component of the wind speed and the speed of the lateral displacement of the projectile, which is raised to a certain power.

The magnitude of the values of the lateral displacement of the projectile under the action of the crosswind when shooting at short distances, determined based on the proposed mathematical model, slightly differ from the values of the lateral displacement specified in the firing tables.

However, with increasing firing distance, the difference between these values is constantly increasing and the value of the lateral displacement of the projectile determined theoretically is much larger than indicated in the firing tables. In addition, in this research the influence of the tank velocity on the value of the projectile lateral displacement taking into account the action of the crosswind is studied.

***Keywords**: external ballistics of the projectile, Makh number, crosswind,cannon D-81, firing tables.*

UDC 623.4:629.36 DOI: https://doi.org/10.33577/2312-4458.25.2021.29-37

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DYNAMIC FUSION OF IMAGES FROM THE VISIBLE AND INFRARED CHANNELS OF SIGHTSEEING SYSTEM BY COMPLEX MATRIX FORMALISM

The employment of new mathematical and computer approaches for the fusion of target images from the visible and infrared channels of the sightseeing system (SSS) is one of the ways to increase the efficiency of the SSS of armored vehicles. Modern approaches to improving the efficiency of image fusion are aimed to increase the visibility of the target via improving the quality indices of fused images. This paper proposes a fundamentally new approach to image fusion, namely dynamic image fusion, at which the target is observed in the mode of a video clip composed of a sequence of stationary fused images obtained at different parameters of fusion, in contrast to traditional stationary image fusion, at which the decision is made from one fused image. Unlike stationary image fusion, aimed to increase the visibility of the target, the dynamic image fusion allows one to enhance the conspicuity of the target. The principle of dynamic image fusion proposed in this paper is based on matrix formalism, in which the fused image is constructed in the form of a complex vector function, which by its mathematical form is analogous to the Jones vector of elliptically polarized light wave, which in turn opens the possibility of matrix transformation of the complex vector of the fused image and consequently its parameterization by analogy with the Jones matrix formalism for the light wave. The article presents mathematical principles of matrix formalism, which is the basis for dynamic image fusion, gives examples of stationary and dynamic image fusion by the method of complex vector function and compares with the corresponding images, fused by algorithms of weight addition in the field of real and complex scalars. It is shown that by selecting weight coefficients, the general form of a complex amplitude vector image can be reduced to the algorithms of weight and averaged addition in the field of real scalars, weight amplitude and RMS-image in the field of complex scalar numbers, and geometric-mean image in the field of complex vectors, which, thereby, are partial cases of the general form of the complex amplitude image in the field of complex vectors.

The animated images obtained by the method of complex vector function illustrate the increase of conspicuity of the object of observation due to the dynamic change of the fusion parameters.

***Key words**: digital image processing, image fusion, infrared imaging, Jones matrix formalism, complex vector image fusion*

Introduction

Analysis of recent publications and aim of the paper. Improvement of armored weapons can be achieved via a variety of approaches. The enhancement of the efficiency of sightgseeing systems (SSS) is among them and in turn, is achieved via a technical improvement of the material component (hardware) of the SSS [1, 2] as well as via the development and application of new mathematical and computer approaches for registration, and fusion of target images obtained from the visible and infrared channels of SSS [3, 4,]. Modern approaches to the improvement of the efficiency of image fusion aimed to increase the quality indices of fused images, which per se reduces to the enhancement of the visibility of the target [5]. This paper presents a principally new approach to image fusion, namely the dynamic image fusion, at which the target is observed in the mode of a video clip assembled of the set of stationary fused images obtained at different parameters of the fusion, unlike traditional stationary image fusion, at which the decision-making is based on the analysis of a single one fused image (even though possibly of the best quality) image. Importantly, at a traditional stationary image fusion, the decision on the best-fused image is produced by the analysis of a set of single fused images, obtained at different fusion parameters.

The advantages of dynamic fusion image are based on the enhancement of the conspicuity of the target in contrast to the increase of its visibility at the stationary image fusion. The introduction of the term conspicuity is the modern concept [6] in the problem of visual search [7-11] in the framework of the theory of digital image processing [12, 13]. Obviously, increasing conspicuity reduces target retrieval time and increases the probability of target detection, recognition, and identification. However, the criteria of conspicuity and the corresponding numerical index of conspicuity, and hence the analytical dependence of the targeting task performance (TTP) on the target conspicuity remain to be open questions.

It should be noted that the conspicuity and visibility of the target are not equivalent terms. The crew commander and the tank gunner are interested not so much in the visibility of the target as in its conspicuity. But the latter does not mean that there is no relation between the two terms. High visibility is necessary, but not a sufficient condition for at least sufficient conspicuity of the target. But conspicuity is a sufficient condition for high visibility of the target. In other words, a conspicuous target is clearly visible, but not vice versa: a well-visible target is not necessarily conspicuous. As an argument in favor of this statement, it is worth recalling the popular phrase: "If you want to hide a thing, put it in a trivially observable place." This statement illustrates that the conspicuity of a target is governed not only by its visibility, but also by many other factors, including purely psychological such as the ability to attract attention and purely physical, such as background homogeneity, i.e. spatial homogeneity of contrast, but not only. Some of the factors are difficult to attribute to purely psychological or purely physical. In particular, the conspicuity of an object is sometimes defined as a property opposite to lateral masking [14- 17], which is a psycho-physical effect unlike the visibility, which is defined by the physical properties of the object and environment (normalized contrast), characterizing the target-background situation. The essence of lateral masking is that an object that is clearly visible on a uniform background becomes inconspicuous when surrounded by many other similar objects or details of a variegated external environment. However, a low-visibility target in a variegated environment becomes well-conspicuous if its visibility changes (modulated) over time, for example by changing its brightness. It is well known [18-20] that a pulsed or modulated light signal is much more conspicuous than a constant (in time) signal of the same brightness. This technique is used, for example, to increase the conspicuity of road signs via their flashing or the visibility of advertising signs via animation effects. The very enhancement of the conspicuity of the target via the animated modulation of its visibility and is the advantage of dynamic image integration in comparison with traditional stationary integration.

The principle of dynamic image fusion proposed in this paper is based on the matrix formalism proposed by us in the previous work [21] based on image fusion in the form of a complex vector function, which in its mathematical form is analogous to the Jones vector of an elliptically polarized light wave. The mathematical similarity of the complex vector of the complex image and the Jones vector opens the possibility of matrix transformation of the complex vector of the fused image and its parameterization by analogy with the Jones matrix formalism for the light wave. Consequently, the fused image can be composed in the form of a video, in which the fused image changes animately with a smooth change of the fusion parameters. In other words, to increase the conspicuity of the target, instead of stationary fusion, in which a single fused image is composed of two input (visible and infrared) images, the dynamic fusion is proposed, in which a fused video image is composed of two input images by animating the fusion parameters.

The principles of matrix formalism, which is the base for the dynamic fusion, are presented in the next (first) section of the paper. The second section presents examples of stationary and dynamic fusion by the method of a vector function and compares with the corresponding images, fused by the algorithms of weighted addition of real and complex scalars.

Results

1. The principle of dynamic fusion by the method of complex vector function

According to the fusion method of a complex vector function, one of the two input images (visible u or infrared υ) is selected as real and the other one (infrared υ or visible u) as the imaginary component of a complex two-dimensional vector $\vec{\psi}$ [21]. Since there are no restrictions on which of the two images to choose for the real and which for the imaginary component, in the general case, one can form two complex vectors

$$\vec{\psi}^{(0)}_{neg} = \frac{1}{\sqrt{2}}\begin{bmatrix} u \\ i\upsilon \end{bmatrix} \quad (1)$$

$$\vec{\psi}^{(0)}_{pos} = \frac{1}{\sqrt{2}}\begin{bmatrix} \upsilon \\ iu \end{bmatrix} \quad (2)$$

which correspond to two complex vector images. Subscripts *neg* and *pos* correspond to negative and positive images in the usual sense of these terms in the image theory. The explanation of this statement is similar to how it is introduced in the method of complex scalar function [3, 4] for fusion of visible and infrared images. It should be noted that $\vec{\psi}^{(0)}_{neg}$ and $\vec{\psi}^{(0)}_{pos}$ specify vectors in a certain vector basis of two-dimensional unit vectors $\vec{c}_1$ and $\vec{c}_2$ so that equations (1) and (2) can be written in the form

$$\vec{\psi}^{(0)}_{neg} = \frac{1}{\sqrt{2}}\left(u\vec{c}_1 + i\upsilon\vec{c}_2\right) \quad (3)$$

$$\vec{\psi}^{(0)}_{pos} = \frac{1}{\sqrt{2}}\left(\upsilon\vec{c}_1 + iu\vec{c}_2\right) \quad (4)$$

The superscript (0) in equations (1) - (4) indicates that the vectors $\vec{\psi}^{(0)}_{neg}$ and $\vec{\psi}^{(0)}_{pos}$ are defined in the Cartesian coordinate system, in which the coordinate axes are directed along the vectors $\vec{c}_1$ and $\vec{c}_2$. Due to the similarity of equations (1), (2) to the mathematical form of the Jones vector of an elliptically polarized light wave by analogy for vectors $\vec{\psi}^{(0)}_{neg}$ and $\vec{\psi}^{(0)}_{pos}$ one can introduce the notions of amplitude $\left|\vec{\psi}^{(0)}_{neg,pos}\right|$, angles of the azimuth $\chi^{(0)}_{neg,pos}$ and ellipticity $\gamma^{(0)}_{neg,pos}$, the algorithms for calculating of which are explicitly presented in [21]. In its own Cartesian coordinate system (with axes along the vectors $\vec{c}_1$ and $\vec{c}_2$) one finds

$$\left|\vec{\psi}^{(0)}_{neg}\right| = \left|\vec{\psi}^{(0)}_{pos}\right| = \frac{1}{\sqrt{2}}\sqrt{u^2+\upsilon^2} \quad (5)$$

$$\tan\chi^{(0)}_{neg,pos} = 0 \quad (6)$$

$$\sin 2\gamma^{(0)}_{neg} = \sin 2\gamma^{(0)}_{pos} = 2\frac{u\upsilon}{u^2+\upsilon^2} \quad (7)$$

To compare the fusion methods of the complex vector function considered in this paper and a scalar function developed in [3,4], we note that for normalized scalar functions

$$\psi_{neg} = \frac{1}{\sqrt{2}}\left(u + i\upsilon\right) \quad (8)$$

$$\psi_{pos} = \frac{1}{\sqrt{2}}\left(\upsilon + iu\right) \quad (9)$$

which are defined in the field of complex scalar numbers, one can calculate the amplitude

$$\left|\psi_{neg}\right| = \left|\psi_{pos}\right| = \frac{1}{\sqrt{2}}\sqrt{u^2+\upsilon^2} \quad (10)$$

and phase

$$\varphi_{neg} = \arctan\frac{\upsilon}{u} \quad (11)$$

$$\varphi_{pos} = \arctan\frac{u}{\upsilon} \quad (12)$$

or the tangent of phase, respectively

$$t_{neg} = \tan\varphi_{neg} = \frac{\upsilon}{u} \quad (13)$$

$$t_{pos} = \tan\varphi_{pos} = \frac{u}{\upsilon} \quad (14)$$

Equations (5) and (10) show that the amplitudes of the vector function, Eq. (5), in the own coordinate system and the scalar function, Eq. (10), coincide. Other algorithms, namely: azimuth and ellipticity angles for a vector function and the phase for a scalar function are different.

An important advantage of vector function image fusion is that vectors $\vec{\psi}^{(0)}_{neg}$ and $\vec{\psi}^{(0)}_{pos}$, defined in the own coordinate system by equations (1) and (2) can be transformed by the action of complex 2×2 matrix J, by transformation,

$$\vec{\psi}_{neg,pos} = J\vec{\psi}^{(0)}_{neg,pos} \quad (15)$$

which is analogous to the matrix transformation in Jones formalism, which is used to describe the propagation of a light polarized wave through an anisotropic optical (in the general case: absorbing) medium [22-25]. Under the action of the matrix J the vectors $\vec{\psi}^{(0)}_{neg}$ and $\vec{\psi}^{(0)}_{pos}$ transform into vectors $\vec{\psi}_{neg}$ and $\vec{\psi}_{pos}$, amplitudes and angles of azimuth and ellipticity of which differ from those given by equations (5) - (7) for vectors $\vec{\psi}^{(0)}_{neg}$ and $\vec{\psi}^{(0)}_{pos}$. In the general case, the amplitudes of the transformed vector $\vec{\psi}$, composed of the input visible u and infrared υ images are of the form:

$$\left|\vec{\psi}_{neg,pos}\right| = \sqrt{a^2_{neg,pos}u^2 + b^2_{neg,pos}\upsilon^2 + c^2_{neg,pos}u\upsilon} \quad (16)$$

where parameters $a^{\psi}_{neg,pos}$, $b^{\psi}_{neg,pos}$ та $c^{\psi}_{neg,pos}$ are composed of elements of the matrix J; their explicit forms can be found in [21]. It follows from Eq. (16) that by their physical sense the parameters $a^{\psi}_{neg,pos}$, $b^{\psi}_{neg,pos}$

and $c^{\psi}_{neg,pos}$ are weight coefficients. For practical purposes, one can abstract from their explicit analytical forms as functions of matrix elements J. It is sufficient to take into account that in essence, they are weight coefficients that can be changed within certain limits according to the norming condition. Taking into account that the vectors $\vec{\psi}^{(0)}_{neg}$ and $\vec{\psi}^{(0)}_{pos}$ describe a complex image, their amplitudes can be normalized to 1. This means that at maximum values $u^{\max} = \upsilon^{\max} = 1$ for coefficients $a^{\psi}_{neg,pos}$, $b^{\psi}_{neg,pos}$ and $c^{\psi}_{neg,pos}$ the following relation holds

$$\left(a^{\psi}_{neg,pos}\right)^2 + \left(b^{\psi}_{neg,pos}\right)^2 + \left(c^{\psi}_{neg,pos}\right)^2 = 1 \qquad (17)$$

In the general case, all three weight oefficients can be varied independently. In partial cases, one can select convenient combinations between them. For example, taking into account equation (17), one can set any two of the three coefficients independent such that their running values are within the intervals $[0;1]$ and $[1;0]$, respectively, and then the value of the third coefficient will be determined from Eq. (17). If the fused image given by Eq. (16) is considered in dynamic mode, i.e., such that the coefficients $a^{\psi}_{neg,pos}$, $b^{\psi}_{neg,pos}$ run all real values between 0 and 1, then the indices *neg* and *pos* can be omitted, because when the coefficients continuously vary, the negative and positive images smoothly transform into each other. Therefore, equation (16) can be rewritten as

$$|\vec{\psi}| = \sqrt{a^2u^2 + b^2\upsilon^2 + c^2u\upsilon} \qquad (18)$$

Taking into account condition (17), Eq. (18) can be rewritten in the form

$$|\vec{\psi}| = \sqrt{a^2u^2 + b^2\upsilon^2 + \left(1 - a^2 - b^2\right)u\upsilon} \qquad (19)$$

From equation (19) it is seen that when $a=1$, $b=0$, one has $c=0$ and then the amplitude of the fused complex vector image reduces into the visible image $|\vec{\psi}|_{a=1,b=0} = u$. At $b=1$, $a=0$ and consequently $c=0$, one has an infrared image $|\vec{\psi}|_{b=1,a=0} = \upsilon$. At $a=b=0$ one has $c=1$ and one obtains the geometric mean of the input images u and υ

$$|\vec{\psi}|_{a=b=0} = \sqrt{u\upsilon} \qquad (20)$$

Obviously, by the selection of coefficients *a*, *b*, *c* one can also obtain the root mean square (RMS) of the input images u and υ. Note that the RMS image, which is obtained as a partial case of the amplitude vector image given by Eq. (18) or its normalized form, i.e. Eq (19), when $a = b = 1/\sqrt{2}$ and as a result $c=0$, coincides with the normalized amplitude image in the field of complex scalars and is of the form

$$|\vec{\psi}|_{a=b=1/\sqrt{2},c=0} = |\psi| = \frac{1}{\sqrt{2}}\sqrt{u^2 + \upsilon^2} \qquad (21)$$

Taking $c^2 = 2ab$, one obtains the weighted sum of the input images in the field of real scalars

$$|\vec{\psi}|_{c^2=2ab} = au + b\upsilon \qquad (22)$$

with appropriate rationing (norming) $a+b=1$. At $a=b=1/2$ one has $c^2 = 1/2$ and Eq. (22) transforms into the arithmetic mean of the input images in the field of real scalars

$$|\vec{\psi}|_{a=b=c^2=1/2} = \frac{u+\upsilon}{2} \qquad (23)$$

Equation (21) is nothing else but a normalized amplitude of a scalar complex function defined by Eqs. (8), (9), which is an amplitude algorithm for image fusion by the method of a complex scalar function [3,4], i.e. in the field of complex scalars. Equation (23) specifies the form of a fused image by the averaging method in the field of real scalars. Thus, the fused images defined by the arithmetic mean (Eq. (23)), RMS (Eq. (21)) and geometric mean (Eq. (20)) algorithms are partial cases of the amplitude algorithm (18) of image fusion in the field of complex vectors.

An animated image composed via continuous variation of weight coefficients *a*, *b*, *c* (or coefficients *a*, *b* using condition (17)) reflects the essence of dynamic fusion by amplitude algorithm. Similarly, one can perform image fusion by the algorithms of azimuth χ and ellipticity γ angles. This consideration will be the subject of our next work. Transformation of azimuth χ and ellipticity γ angles under the action of complex 2×2 matrix *J*, which is analogous to the Jones matrix for the light wave, is presented explicitly in our paper [21].

Thus, dynamic image fusion in the field of complex vectors enables the comparison of images fused at different values of weight coefficients and to select the most informative of them. In addition, the dynamic nature of image fusion, in which the visibility of the object of observation (target) changes over time increases its conspicuity.

In the next section, we present stationary fused images obtained by the weight amplitude algorithm of a complex vector function and its partial cases: arithmetic avarege, RMS, and geometric mean, as well as dynamic (animated) amplitude images obtained in the field of real scalars, complex scalars, and complex vectors.

2. Results and discussion

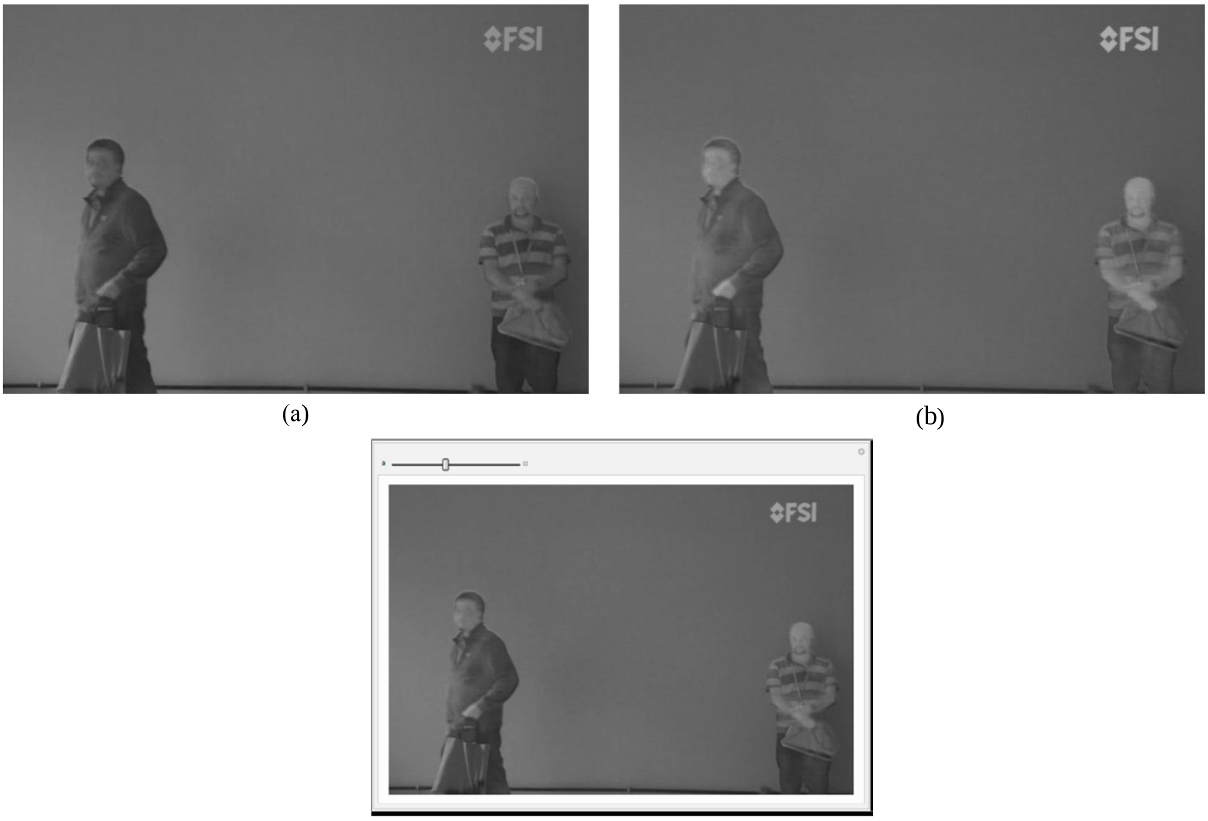
Input visible u and infrared υ the images are shown in Fig. 1, which shows two men with packages in their hands. In the visible image (Fig. 1a) their faces are clearly visible, but the contents of the packages they hold in their hands are not visible. In the infrared image (Fig. 1b), on the other hand, it is clear that the man on the left has an object resembling a gun in the package, and the man on the right has an object resembling a knife in the package, but their faces are hardly recognizable.

Weight addition of images in the field of real scalars according to the algorithm defined by Eq. (22) at $a = 0.3, b = 0.7$ (Fig. 2a) and average addition (Fig. 2b) by algorithm (23) lead to the conclusion that the fused image by its quality (contrast) is worse than both visible (Fig. 1a) and IR image (Fig. 1b).

(a) (b)

Fig. 1. **Input (a) visible** u **and (b) infrared** υ **images** [26]

Dynamic image fusion by smooth variation of weight parameters *a* and *b* according to the algorithm of weight addition (22) also illustrates this conclusion (Fig. 2c). The faces of people and/or objects hidden in their bags are hard to see in the images fused in the field of real and complex scalars. This result is consistent with the conclusion drawn in [27] that the contrast of the image, fused by the algorithm of weight addition is always worse than the contrast of one of the input images.



(a) (b)

(c)

Fig. 2. **Weight addition in the field of real scalars according to the algorithm defined by Eq. (22) with the norming condition** $a+b=1$ **at (a)** $a = 0.3$ **, (b)** $a = 0.5$ **, corresponding to the arithmetic mean of the algorithm given by equation (23) and dynamic fusion by weight algorithm at** $a\in[0;1]$ **(c). To view the video, click by the mouse cursor on the video**

The RMS image (Fig. 3a), obtained by the amplitude algorithm (21) of the vector complex function, Eqs. (1) and (2), coincides with the amplitude algorithm (10) of the complex scalar function [3,4] and visually hardly differs from the arithmetic mean image (Fig. 2b), while on the geometric mean image, fused by the algorithm (20) both faces of persons and the contents of their bags are clearly visible (Fig. 3c).

Corresponding dynamic images, fused by the weight amplitude algorithm $w=\sqrt{(1-a)^2u^2+a^2\upsilon^2}$ of the complex scalar function method varying the weight parameter in the range $a \in [0;1]$ is shown in Fig. 3c, and the dynamic image obtained by the amplitude algorithm (18) of the complex vector function is shown in Fig. 3d.

Two important conclusions follow from the visual analysis of the fused images presented in Fig. 2 and 3. First, among the stationary images obtained by different methods, the most informative is the geometric mean image (Fig. 3b), obtained by the method of a complex vector fusion. Fig. 3b clearly displays both the faces of people and objects (a gun in one and a knife in the other) in packages in their hands. Secondly, the application of the principle of dynamic fusion allows us to demonstrate that the very these individuals hide in their packages the very such firearms and bladed weapons. The use of dynamic fusion enhances the conspicuity of objects of observation (hidden weapons in packages) by the variation of the visibility of hidden weapons from zero in the visible image (in the video image, Fig. 3d, displayed using Eq. (19) at $a = 1,\ b = 0$) to such that is clearly visible on the IR image (at $b = 1,\ a = 0$). The visibility of the faces changes in the opposite direction from good visibility in the visible image to poor visibility in the IR image. It is clear that the visual analysis of the stationary images corresponding to different values of weight parameters *a* and *b* would take much longer time than watching a video clip of images fused at different values *a* and *b*

In fact, visual analysis of every single stationary image requires focusing on each of them, time to assess the quality of each of them, time to switch attention to each subsequent image, and time to compare them. If we assume that one stationary complex image will take on average time t_c, then for viewing n single images one needs time $t = nt_c$. To watch a video composed of these n images one needs approximately the same time as for one single stationary image, i.e. t_c. In this case, to watch the video one needs the time, which is by n times less than that to view n stationary fused images.

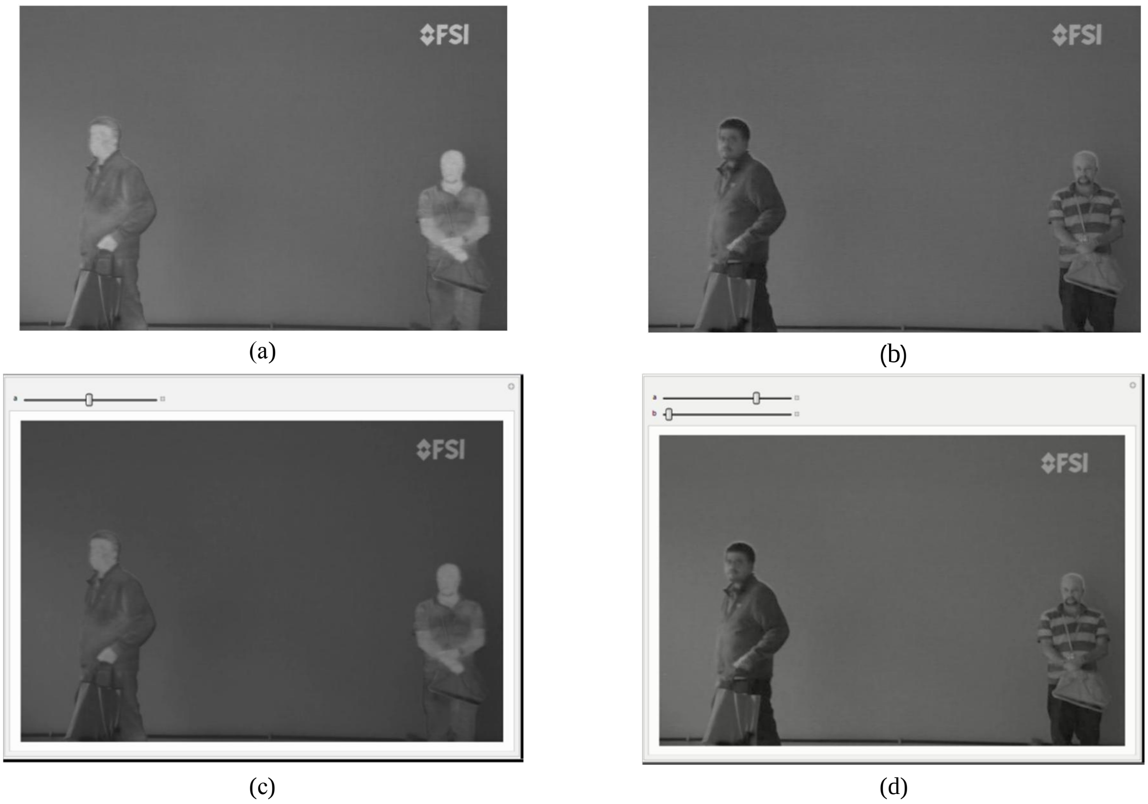


Fig. 3. **Stationary RMS (a), geometric mean (b) fused images and the corresponding dynamic video images by weight amplitude algorithms in the field of scalar complex numbers (c) and complex vectors (d). To view the video, click the mouse cursor on the corresponding video**

This is easy to see: watching the video presented in Fig. 3d takes about 5 seconds. This video is composed by varying the parameters a and b in the range from 0 to 1. If we assume that each of the parameters changes with a step of 0.1, then such a step implies 20 (10 for each parameter) stationary images. On average it takes at least 5 seconds (and possibly more) to view one stationary image. In this case, watching the video takes time, which is less by at least 20 (and possibly more) times than watching the 20 stationary fused images, from which the video is composed. For a smaller step variation of the parameters a and b, for example, 0.01, the number of stationary images increases 10 times, i.e. one has $n = 200$. The video duration can remain unchanged (video viewing speed can be adjusted). In this case, the video viewing time is 200 times shorter than the time to view stationary images.

Conclusions

Improving the efficiency of the visual task of the target information acquisition can be achieved by enhancing the conspicuity of the target. Improving the conspicuity of the target is the modern concept in the task of visual search and data acquisition on the target. Conspicuity of the target is a more capacious generalized concept in comparison with the concept of visibility of the target. The visibility of the target as a numerical parameter of image quality is analytically expressed through the contrast of the target and characterizes the target-background situation, while the visibility of the target is a psycho-physical concept that reflects the ability to draw attention to the target at its given visibility. High visibility of the target is necessary, but not a sufficient condition for the conspicuity of the target. Improving the conspicuity of the target can be achieved in various ways. Animation effects are among them.

Dynamic image fusion in the form of a video clip composed of stationary fused images obtained at different fusion parameters is the basis for the animated enhancement of the conspicuity of the object of observation, proposed in this work. Dynamic fusion is provided within the method of image fusion from the visible and IR spectral ranges in the field of complex two-dimensional vectors. Namely, from the input visible u and infrared υ images one forms a complex two-dimensional vector $\vec{\psi}_0$ so that one of the input images is selected for its real, and the other for its imaginary component. In such a way the vector $\vec{\psi}_0$ mathematically has the form of a two-dimensional Jones vector of elliptical-polarized light wave. Using this analogy, one can apply Jones's matrix formalism to construct a transformed vector of a fused image $\vec{\psi} = J\vec{\psi}_0$, where J is a matrix of size 2×2, composed of complex elements, analogous to the Jones matrix used to describe an optically anisotropic medium. By analogy with Jones matrix formalism, the input visible and IR images can be fused by the amplitude algorithm, and the algorithms of azimuth and ellipticity. In this paper, we focus on the amplitude algorithm. In the general case of nonzero all complex components of the matrix J the amplitude of the complex fused image is of the form $|\vec{\psi}| = \sqrt{a^2u^2 + b^2\upsilon^2 + c^2 u\upsilon}$. Parameters *a*, *b*, *c* are combinations of matrix elements *J* and play the role of weight coefficients that vary from 0 to 1 and satisfy the rationing (also called norming) condition $a^2 + b^2 + c^2 = 1$. As a result, changing the weights *a*, *b* in the range from 0 to 1 with a certain step and taking $c^2 = 1 - a^2 - b^2$ of the two inputs (visible u and infrared υ) images one can generate a set of complex images. Having formed a video file from this set of images, we obtain a dynamic fused image. It is the possibility of varying the weight coefficients that is the basis for a dynamic image fusion within the method of a vector complex function by the Jones matrix formalism.

By the appropriate setting of weight coefficients, the general form of the complex amplitude vector image can be reduced to algorithms of weight and averaged addition in the field of real scalars, weight amplitude, and RMS square image in the field of complex scalar numbers and geometric mean image in the field of complex vectors, which are thus partial cases of the general form of the complex amplitude image in the field of complex vectors.

The animated images obtained by the method of complex vector function illustrate the enhancement of conspicuity of the object of observation due to the dynamic variation of the fusion parameters.

Acknowledgement: The Authors are grateful to the team of the SMT/COPPE/Poli/UFRJ and IME–Instituto Militar de Engenharia within the CAPES/Pró–Defesa Program, in a partnership with IPqM–Instituto de Pesquisa da Marinha for the development and kind permission for free downloading of the partial Vis– and IR–images from the Visual and Infrared Database image Fusion [26].

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Динамічне комплексування зображень з видимого та інфрачервоного каналів прицільно-спостережного комплексу за методом комплексного матричного формалізму

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Застосуванням нових математичних та комп'ютерних підходів для реєстрації та комплексування зображень цілей із видимого та інфрачервоного каналів прицільно-спостережного комплексу (ПСК) – один із шляхів підвищення ефективності ПСК зразків бронетанкового озброєння. Сучасні підходи до підвищення ефективності комплексування зображень зводяться до підвищення видимості цілі шляхом підвищення індексів якості комплексованих зображень. В роботі обґрунтовується принципово новий підхід до комплексування зображень, а саме динамічне комплексування, при якому ціль спостерігається у режимі відеокліпа, скомпонованого із послідовності стаціонарних комплексованих зображень, отриманих при різних параметрах комплексування, на відміну від традиційного стаціонарного комплексування, при якому рішення приймається із одного комплексованого зображення. На відміну від стаціонарного комплексування, метою якого є підвищення видимості цілі, динамічне комплексування дає змогу підвищити помітність цілі. Принцип динамічного комплексування, що пропонується в роботі, ґрунтується на матричному формалізмі, при якому комплексоване зображення будується у формі комплексної векторної функції, що за своєю математичною формою є аналогом вектора Джонса еліптично поляризованої світлової хвилі, що відкриває можливість матричного перетворення вектора комплексного комплексованого зображення та його параметризації за аналогією матричного формалізму Джонса для світлової хвилі. В статті наведено математичні принципи матричного формалізму, який лежить в основі *динамічного комплексування, наведено приклади стаціонарного та динамічного комплексування за методом векторної функції та проведено порівняння із відповідними зображеннями, комплексованими за алгоритмами вагового складання в полі дійсних та комплексних скалярів. Показано, що підбором вагових коефіцієнтів загальний вигляд комплексованого амплітудного векторного зображення можна звести до алгоритмів вагового та усередненого складання в полі дійсних скалярів, вагового амплітудного та середньоквадратичного зображення в полі комплексних скалярних чисел та середньогеометричного зображення в полі комплексних векторів, які, таким чином, є частковими випадками загальної форми комплексованого амплітудного зображення в полі комплексних векторів.*

Наведені в роботі анімаційні зображення, отримані за методом комплексної векторної функції, наочно ілюструють підвищення помітності об'єкта спостереження за рахунок динамічної зміни параметрів комплексування.

Ключові слова: *цифрова обробка зображень, комплексування зображень, тепловізійне бачення, формалізм матриць Джонса, комплексування зображень методом комплексної векторної функції.*

Динамическое комплексирование изображений видимого и тепловизионного каналов прицельно-наблюдательного комплекса методом матричного формализма

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Использование новых математических и компьютерных подходов комплексирования изображений целей, полученных из видимого и инфракрасного каналов прицельно-наблюдательного комплекса (ПСК) - один из способов повышения эффективности ПСК образцов бронетехники. Современные подходы к повышению эффективности комплексирования изображений направлены на повышение заметности цели за счет улучшения показателей качества комплексированных изображений. В статье предлагается принципиально новый подход к комплексированию изображений, а именно – динамическое комплексирование изображений, при котором цель наблюдается в режиме видеоклипа, составленного из последовательности стационарных объединенных изображений, полученных при различных параметрах объединения, в отличие от традиционного стационарного комплексирования, при котором решение принимается по одному комплексированному изображению. В отличие от стационарного комплексирования, направленного на увеличение видимости цели, динамическое объединение изображений позволяет повысить заметность цели. Предлагаемый в статье принцип динамического объединения изображений основан на матричном формализме, в котором комплексированное изображение строится в виде комплексной векторной функции, которая по своей математической форме аналогична вектору Джонса эллиптически поляризованной световой волны, который в свою очередь открывает возможность матричного преобразования комплексного вектора объединенного изображения и, следовательно, его параметризации по аналогии с формализмом матрицы Джонса для световой волны.В статье представлены математические основы матричного формализма, лежащего в основе динамического объединения изображений, приведены примеры объединения стационарных и динамических изображений методом комплексной векторной функции и проведено сравнение с соответствующими изображениями, объединенными алгоритмами весового сложения в области действительных и комплексных скаляров. Показано, что путем выбора весовых коэффициентов общий вид векторного изображения комплексной амплитуды может быть сведен к алгоритмам весового и усредненного сложения в области вещественных скаляров, весовой амплитуды и RMS-изображения в области комплексных скалярных чисел, и среднегеометрическое изображение в области комплексных векторов, которые, таким образом, являются частными случаями общего вида комплексного амплитудного изображения в области комплексных векторов.

Анимированные изображения, полученные методом комплексной векторной функции, иллюстрируют увеличение заметности объекта наблюдения за счет динамического изменения параметров слияния.

Ключевые слова: *цифровая обработка изображений, комплексирование изображений, инфракрасное изображение, формализм матриц Джонса, комплексирование изображений методом комплексной векторной функции.*