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GENERALISED SURVIVABILITY MODEL FOR ARMOURED WEAPONS AND EQUIPMENT

T. Stakh, R. Sidor, D. Khaystov, Ya. Khaystov, O. Kyrychuk, V. Mudryk, Yu. Nastishin

Today, there is no single, standardized approach to quantifying the survivability of BTOT samples. The survivability models available in the literature differ in the number of components that define the essence of the concept of survivability, depending on the type of military equipment and conditions of its use. The analysis of the literature shows that the following components of the survivability of BTOT include at least the following: 1) secrecy, 2) ability to eliminate the threat by own means, 3) security, 4) recoverability, 5) mobility, 6) resistance to overturning, 7) resistance to spontaneous operational damage or own defects. In order to take into account, the contribution of survivability components to the overall survivability of BTOT, the paper develops the theoretical basis for their classification as random additive/multiplicative statistical events. The proposed step-by-step algorithm for classifying survivability components as random events makes it possible to determine the rule by which their probabilities are combined into the overall survivability indicator. In the available information sources, the above seven components of survivability are considered in various combinations, but the simultaneous consideration of all seven components has not yet been carried out. Moreover, an algorithm for taking into account new components of resilience, if any, remains to be developed. This paper is devoted to the development of a generalized model for assessing durability, taking into account the seven known durability components and with the possibility of introducing new components based on the classification of durability components as random events. The proposed generalized model of sample survivability is developed in two stages. The first stage is to create a basic survivability model that takes into account the seven generally accepted components. At the second stage of developing the generalized model, we propose a new survivability component, which we call threat activity, and use its example to illustrate how new components can be added to the basic model. To quantify the effectiveness of the resilience model and the efficiency of innovative measures to improve specific resilience components, we propose appropriate quantitative indicators.

Keywords: *survivability of armoured weapons and equipment; operational and combat survivability of military equipment; components of military equipment survivability; additive and multiplicative random events.*

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V. Hrabchak, A. Kosovtsov, V. Hrabchak

Hetman Petro Sahaidachnyi National Army Academy, Lviv

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IDENTIFICATION OF NONLINEAR AERODYNAMIC PROJECTILE COEFFICIENTS BASED ON A MODIFIED POINT MASS TRAJECTORY MODEL

To increase calculation of projectile flight trajectories effectiveness based on mathematical models which describe the spatial movement of the projectile in the air, a pressing question it is relevant to determine individual aerodynamic projectile coefficients with specified accuracy. The construction of modern mathematical models of projectile flight is based on an approximate approach which is called the small-angle approximation. According to this approach, the aerodynamic coefficients are expanded in a Taylor series in terms of the angle of attack and only the linear terms of the expansion are retained, which allows to significantly simplify the mathematical models of the projectile's flight, but considerably worsens the accuracy of calculating its flight trajectories. The most suitable for determining the aerodynamic coefficients of a projectile is a modified point mass trajectory model, as a mathematical model of projectile flight (STANAG 4355 (Edition 3)). The article presents procedures for converting a modified point mass trajectory model into a system of differential-algebraic equations provided in the real form, which, given the appropriate set of linear and nonlinear aerodynamic coefficients, allows calculating the main parameters of the projectile's flight with less computational resources. Analytical expressions were obtained for identifying the aerodynamic coefficients of the drag force, lift force, Magnus force, decreasing of the projectile's *rotational speed, its overturning moment, and the square of the module of the projectile's angle of attack. It is shown that, given a known function of the change in the angular velocity of the projectile's own rotation, the obtained analytical expressions functionally depend exclusively on the parameters obtained from external trajectory measurements (projectile flight coordinates and their derivatives).*

***Key words: projectile,** aerodynamic force, linear and non-linear aerodynamic coefficients, mathematical model, modified model, identification, flight parameters, yaw of repose.*

Introduction

General statement of the problem and analysis of the literature. The mathematical basis of external ballistics is mathematical models (MM) of the projectile flight dynamics which connect the kinematic characteristics of the projectile (coordinates, the velocity of the center of mass (CM) and its position in space, described by the yaw of repose of the projectile axis relative to the velocity vector) with the forces acting on the projectile.

The most accurate MM today is considered to be the so-called model with six degrees of freedom – 6DOF [1, 2]. 6DOF model, based on the theory of motion of a perfectly rigid body, developed by Euler in the 18th century, appeared in ballistics at the beginning of the last century and continues to develop to this day. In the theory of motion of a perfectly rigid body it is assumed that all forces and moments acting on the body are known. However, in external ballistics, one of the most important unsolved problems is the measurement (finding) of individual (for each type of projectile) aerodynamic coefficients which are included in the equation of projectile flight dynamics. It should be noted that all modern MMs are traditionally based on the so-called small-angle approximation, when it is assumed that the projectile is stabilized in flight and, accordingly, its angles of attack are quite small. The angle of attack is formed during the movement of the projectile in the gun barrel, and is caused by the mismatch of the longitudinal axis of the projectile with the axis of symmetry of the gun barrel, as well as the eccentricity (imbalance) of the weight of the projectile (Fig. 1).

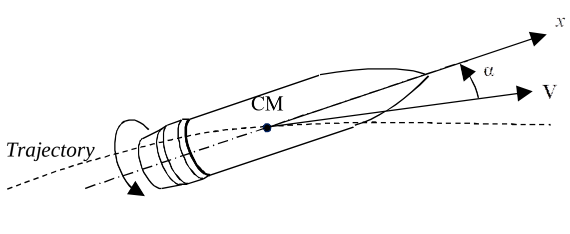

Fig. 1. **Projectile motion on a trajectory**
α – angle of attack of the projectile;
v – projectile velocity vector

The nature and parameters of yaw of repose and their effect on the flight of projectiles are determined by the level of initial disturbances: initial angles and angular velocities of the projectile's yaw of repose oscillations.

It is assumed [1-3] that the forces and moments acting on a projectile moving in air depend only on the Mach number (M) and the square of angle of attack α^2. In addition, this dependence can be represented (expanded) in the form of a Taylor series in powers α^2, i.e.

$$F(M,\alpha^2) = C_0(M) + C_2(M)\alpha^2 + ... + C_{2n}(M)\alpha^{2n} .$$

Limited to n terms, the coefficients of which are called aerodynamic coefficients. This approach reveals the existing “weakness” of the aforementioned models, since a priori the number of retained coefficients of each force or moment is unknown and it is impossible to clearly form the MM for a given projectile. To overcome this “weakness”, it would be more convenient to identify not aerodynamic coefficients $C_i(M)$, but aerodynamic forces (moments) $F(M,\alpha^2)$ and then find aerodynamic coefficients from them.

Determining the aerodynamic coefficients of a projectile is a key stage in the design, modeling, and practical use of its ballistic characteristics. Linear coefficients describe aerodynamic drag and lift at low angles of attack, while nonlinear coefficients take into account more complex phenomena occurring at high angles of attack, including flow disruption and vortex formation. Experimental determination of these coefficients is a complex task that requires the use of specialized methods and equipment.

The choice of the appropriate method is determined by the required accuracy, the available equipment and the range of investigated parameters. Nowadays most common methods are:

ballistic (external-trajectory) methods;
aerodynamic research methods using wind tunnels;
mathematical (numerical) methods.

The wind tunnel balancing method is less accurate for determining linear coefficients, and has certain limitations at large nutation angles. The ballistic method, as a free-flight projectile method, is suitable for studying nonlinear effects, especially under real flight conditions, but its accuracy may be limited by both the accuracy of experimental data and identification methods. However, only the experimental method of free flight of a real projectile can substantiate the accuracy of the results obtained by any methods. Currently, the most popular is a combination of several methods, for example, the use of mathematical methods such as Computational Fluid Dynamics (CFD), wind tunnel data to compare free flight and CFD results, which provides the ability to assess accuracy and reliability [4-14]. However, this approach has a number of significant drawbacks, related both to its extreme mathematical complexity and to the heterogeneity of the source data.

In many of the aforementioned works, the identification of aerodynamic coefficients is carried out based on the 6DOF model, which contains a large number of aerodynamic coefficients - linear and nonlinear, which significantly complicates the identification procedure. It is more appropriate to start the construction and adjustment of effective identification methods on the basis of simpler mathematical models of projectile flight dynamics.

The main model for calculating the flight trajectory of spin-stabilized projectiles in external ballistics is the use of a simpler, compared to the 6DOF model, modified point mass trajectory model (MPMTM) standardized in STANAG 4355, which is also known as the four-degree-of-freedom model, or the R. Leske model [1, 2]. There are several variants of MPMTM in explicit and implicit form [16-19]. MPMTM in explicit form allows us to obtain equivalent equations of projectile flight dynamics, which contain only trajectory characteristics and do not contain characteristics of the angular motion of the projectile. In this case, it is possible to construct a fairly simple and effective method for identifying linear aerodynamic coefficients [20]. However, it is known that in the presence of nonlinear aerodynamic coefficients, reducing the MPMTM to an equivalent explicit form is usually a difficult task.

Therefore, **the aim of the article** is to develop an effective approach to identifying nonlinear aerodynamic coefficients taking into account the new implicit form of MPMTM, equivalent to the original system of equations. On its basis, we obtain algebraic expressions suitable for identification of nonlinear aerodynamic coefficients and the modulus of nutation angles according to the measured trajectory data.

Main body

1. Reduction of the system of differential equations of MPMTM to an equivalent system of differential-algebraic equations. In general form, MPMTM contains the following differential equations [15]:

- equation of motion of the projectile's CM

$$m\dot{\mathbf{u}} = \mathbf{DF} + \mathbf{LF} + \mathbf{MF} + m\mathbf{g} + m\mathbf{\Lambda}; \qquad (1)$$

- equation of rotation of a projectile around its polar axis

$$\dot{p} = \frac{\pi\rho d^4 p \mathrm{v} C_{spin}}{8 I_x}, \qquad (2)$$

with initial conditions

$$p_0 = \frac{2\pi \mathrm{v}_0}{k_c d},$$

where m – projectile mass; $\mathbf{u}$ – the velocity vector of the projectile relative to the Earth's frame of reference; $\mathbf{DF}$ – drag force; $\mathbf{LF}$ – lift (normal) force; $\mathbf{MF}$ – Magnus force; p – angular velocity of the projectile (its own rotational speed); $\mathbf{g}$ – acceleration of gravity; $\mathbf{\Lambda}$ – acceleration from the Coriolis force, which is caused by the Earth's rotation; C_{spin} – the decreasing of the projectile's rotational speed; I_x – polar moment of inertia of the projectile; k_c – relative length of the gun barrel rifling stroke in calibers; d – diameter (caliber) of the projectile; ρ – air density; v_0 – initial velocity of the projectile; p_0 – the angular speed of rotation of the projectile at the cross section of the gun barrel; $\mathbf{v}$ – the velocity vector of the projectile in the terrestrial coordinate system relative to the air, that is determined by the components:

$$\mathbf{v} = [\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_2]; \quad \mathrm{v} = \sqrt{\mathrm{v}_1^2 + \mathrm{v}_2^2 + \mathrm{v}_3^2}. \qquad (3)$$

Here and further in the text of the article, we will adopt the names of the components of the aerodynamic force (moments) in accordance with STANAG 4355 (Edition 3) [15], and we will also use bold letters to denote vectors, and standard letters for scalars.

The drag force and lift force are expressed in vector form as [15]:

$$\mathbf{DF} = -\frac{1}{8}\pi\rho d^2 C_D \mathbf{v}\mathrm{v};$$
$$\mathbf{LF} = \frac{1}{8}\pi\rho d^2 C_L \mathrm{v}^2 \boldsymbol{\alpha}; \qquad (4)$$
$$\mathbf{MF} = \frac{1}{8}\pi\rho d^3 p C_{mag-f} (\boldsymbol{\alpha} \times \mathbf{v}),$$

where C_D, C_L, C_{mag-f} – coefficients of drag force, lift force and Magnus force, respectively.

In vector form, the equation of the angle of attack of the projectile, which is included in the expressions of aerodynamic forces (4), is defined as [15]

$$\boldsymbol{\alpha} = -\frac{8 I_x p (\mathbf{v} \times \dot{\mathbf{u}})}{\pi\rho d^3 \left(C_{M_{\alpha 0}} + C_{M_{\alpha^2}} \alpha^2\right) \mathrm{v}^4}, \qquad (5)$$

where $C_M(M, \alpha^2) = C_M + C_{M_{\alpha^2}}\alpha^2$, $C_M, C_{M_{\alpha^2}}\alpha^2$ – linear and quadratic coefficients of the projectile overturning moment, respectively, with initial conditions

$$\boldsymbol{\alpha_0} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}.$$

The aerodynamic force coefficients C_D, C_L vary depending on the size of the angle of attack of the projectile.

Thus, the drag coefficient is usually approximated by the expression

$$C_D\left(M, \alpha^2\right) = C_{D0} + C_{D_2}\alpha^2,$$

where C_{D0}, C_{D_2} – linear and drag force coefficients, respectively.

The lift coefficient is also characterized by nonlinear behavior and depends on the magnitude of the angle of attack of the projectile.

$$C_L\left(M, \alpha^2\right) = C_L + C_{L_{\alpha^2}}\alpha^2,$$

where C_L, $C_{L_{\alpha^2}}$ – linear and quadratic coefficients of lifting force, respectively.

When considering Magnus forces, one is usually limited to considering only the linear term.

The generalized original system of equations MMPM (1), (5) in scalar form in terms of components in the Cartesian terrestrial coordinate system (Fig. 2) will take the form:

- equation of motion of the projectile's CM.

$$\frac{d}{dt}u_1 = -\frac{1}{2}\frac{\rho SC_D\left(M,\alpha^2\right)\mathrm{v}\mathrm{v}_1}{m} + \frac{1}{2}\frac{\rho SC_L\left(M,\alpha^2\right)\mathrm{v}^2\alpha_1}{m} - -\frac{1}{2}\frac{\rho SdpC_{mag}(M)\left(\alpha_2\mathrm{v}_3 - \alpha_3\mathrm{v}_2\right)}{m}; \quad (6)$$

$$\frac{d}{dt}u_2 = -\frac{1}{2}\frac{\rho SC_D\left(M,\alpha^2\right)\mathrm{v}\mathrm{v}_2}{m} + \frac{1}{2}\frac{\rho SC_L\left(M,\alpha^2\right)\mathrm{v}^2\alpha_2}{m} - -\frac{1}{2}\frac{\rho SdpC_{mag}(M)\left(-\alpha_1\mathrm{v}_3 - \alpha_3\mathrm{v}_1\right)}{m} - g; \quad (7)$$

$$\frac{d}{dt}u_3 = -\frac{1}{2}\frac{\rho SC_D\left(M,\alpha^2\right)\mathrm{v}\mathrm{v}_3}{m} + \frac{1}{2}\frac{\rho SC_L\left(M,\alpha^2\right)\mathrm{v}^2\alpha_3}{m} - -\frac{1}{2}\frac{\rho SdpC_{mag}(M)\left(\alpha_1\mathrm{v}_2 - \alpha_2\mathrm{v}_1\right)}{m}; \quad (8)$$

- equation of yaw of repose of a projectile

$$\alpha_1 = \frac{2I_x p\left(\mathrm{v}_2\frac{d}{dt}u_3 - \mathrm{v}_3\frac{d}{dt}u_2\right)}{\rho SdC_M\left(M,\alpha^2\right)\mathrm{v}^4}; \quad (9)$$

$$\alpha_2 = \frac{2I_x p\left(\mathrm{v}_3\frac{d}{dt}u_1 - \mathrm{v}_1\frac{d}{dt}u_3\right)}{\rho SdC_M\left(M,\alpha^2\right)\mathrm{v}^4}; \quad (10)$$

$$\alpha_3 = \frac{2I_x p\left(\mathrm{v}_1\frac{d}{dt}u_2 - \mathrm{v}_2\frac{d}{dt}u_1\right)}{\rho SdC_M\left(M,\alpha^2\right)\mathrm{v}^4}, \quad (11)$$

where $u_1 = \mathrm{v}_1 + w_1$, $u_2 = \mathrm{v}_2 + w_2$, $u_3 = \mathrm{v}_3 + w_3$ – components of the projectile's flight velocity relative to the Earth's surface; w_1, w_2, w_3 – components of wind speed relative to the Earth's surface.

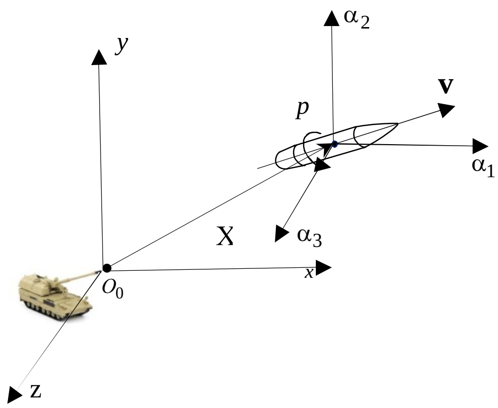

Fig. 2. **Orientation of coordinate systems according to the STANAG 4355 standard**

If we square equations (9)-(11) and sum them, we obtain the equation for the square of the modulus of the angle of attack

$$\alpha^2 = \alpha_1^2 + \alpha_2^2 + \alpha_3^2 = \frac{4I_x^2 p^2\left[\left(\mathrm{v}_2\frac{d}{dt}u_1 - \mathrm{v}_1\frac{d}{dt}u_2\right)^2 + \left(\mathrm{v}_3\frac{d}{dt}u_1 - \mathrm{v}_1\frac{d}{dt}u_3\right)^2 + \left(\mathrm{v}_3\frac{d}{dt}u_2 - \mathrm{v}_2\frac{d}{dt}u_3\right)^2\right]}{\rho^2 S^2 d^2\mathrm{v}^8 C_M^2\left(M,\alpha^2\right)}, \quad (12)$$

where $S_M = \frac{\pi d^2}{4}$ – midsection area (cross-section of the projectile).

The system of differential equations MPMTM (6)-(11) is implicit because the principal derivatives $\dot{u}_i$ simultaneously enter into different equations, which significantly complicates their numerical solution. In works [16-18], a variant of reducing this system to an explicit form is proposed under the assumption that the projectile is described only by linear aerodynamic coefficients, with the exception of the drag coefficient. This significantly simplifies the calculation of projectile flight trajectories and allows for significant progress in the current problem of identifying linear aerodynamic coefficients [20]. However, as experimental studies conducted in the 1960s and 1970s have shown, many projectiles have significant nonlinear aerodynamic coefficients in addition to the drag coefficient [1].

Solving algebraically the system of equations (6)-(11) with respect to $\dot{u}_1, \dot{u}_2, \dot{u}_3, \alpha_1, \alpha_2, \alpha_3$, in the Maple software environment, we obtain in explicit form the system of differential-algebraic equations:

$$\frac{d}{dt}u_1=-\frac{\rho SC_D\left(M,\alpha^2\right)\mathrm{v}\mathrm{v}_1}{2m}-$$
$$-\frac{pgI_x\mathrm{v}_2\left[C_L^2\left(M,\alpha^2\right)I_xp\mathrm{v}^2\mathrm{v}_1-\mathrm{v}_3C_L\left(M,\alpha^2\right)C_M\left(M,\alpha^2\right)dm\mathrm{v}^4+\mathrm{v}_1\mathrm{v}_2\,pd^2C_{mag}(M)\left(m\mathrm{v}^2C_M\left(M,\alpha^2\right)+p^2I_xC_{mag}(M)\right)\right]}{\mathrm{v}^2\left(C_L^2\left(M,\alpha^2\right)I_x^2p^2\mathrm{v}^2+d^2\left(m\mathrm{v}^2C_M\left(M,\alpha^2\right)+p^2I_xC_{mag}(M)\right)\right)}; \quad (13)$$

$$\frac{d}{dt}u_2=-\frac{\rho SC_D(M,\alpha^2)\,\mathrm{v}\mathrm{v}_2}{2m}-$$
$$-\frac{g\left[\mathrm{v}_2^2I_x^2p^2\mathrm{v}^2C_L^2\left(M,\alpha^2\right)+d^2\left(m\mathrm{v}^4C_M\left(M,\alpha^2\right)+\mathrm{v}_2^2p^2I_xC_{mag}(M)\right)\left(m\mathrm{v}^4C_M\left(M,\alpha^2\right)+p^2I_xC_{mag}(M)\right)\right]}{\mathrm{v}^2\left(C_L^2\left(M,\alpha^2\right)I_x^2p^2\mathrm{v}^2+d^2\left(m\upsilon^2C_M\left(M,\alpha^2\right)+p^2I_xC_{mag}(M)\right)\right)}; \quad (14)$$

$$\frac{d}{dt}u_3=-\frac{\rho SC_D(M,\alpha^2)\,\mathrm{v}\mathrm{v}_3}{2m}-$$
$$-\frac{pgI_x\left[\mathrm{v}_2\mathrm{v}_3C_L^2\left(M,\alpha^2\right)I_xp\mathrm{v}^2+\mathrm{v}_1C_L\left(M,\alpha^2\right)C_M\left(M,\alpha^2\right)dm\mathrm{v}^4+\mathrm{v}_2\mathrm{v}_3\,pd^2C_{mag}(M)\left(m\mathrm{v}^2C_M\left(M,\alpha^2\right)+p^2I_xC_{mag}(M)\right)\right]}{\mathrm{v}^2\left(C_L^2\left(M,\alpha^2\right)I_x^2p^2\mathrm{v}^2+d^2\left(m\mathrm{v}^2C_M\left(M,\alpha^2\right)+p^2I_xC_{mag}(M)\right)\right)};$$

(15)

$$\alpha_1=-\frac{2mpgI_x\left[\mathrm{v}_1\mathrm{v}_2\,pI_xC_L\left(M,\alpha^2\right)-d\,\mathrm{v}_3\left(m\mathrm{v}^2C_M\left(M,\alpha^2\right)+p^2I_xC_{mag}(M)\right)\right]}{\rho S\mathrm{v}^2\left[C_L^2\left(M,\alpha^2\right)I_x^2p^2\mathrm{v}^2+d^2\left(m\mathrm{v}^2C_M\left(M,\alpha^2\right)+p^2I_xC_{mag}(M)\right)\right]}; \quad (16)$$

$$\alpha_2=\frac{2mp^2gI_x^2\left(\mathrm{v}_1^2+\mathrm{v}_3^2\right)C_L\left(M,\alpha^2\right)}{\rho S\mathrm{v}^2\left[C_L^2\left(M,\alpha^2\right)I_x^2p^2\mathrm{v}^2+d^2\left(m\mathrm{v}^2C_M\left(M,\alpha^2\right)+p^2I_xC_{mag}(M)\right)\right]}; \quad (17)$$

$$\alpha_3=-\frac{2mpgI_x\left[\mathrm{v}_2\mathrm{v}_3\,pI_xC_L\left(M,\alpha^2\right)+d\,\mathrm{v}_1\left(m\mathrm{v}^2C_M\left(M,\alpha^2\right)+p^2I_xC_{mag}(M)\right)\right]}{\rho S\mathrm{v}^2\left[C_L^2\left(M,\alpha^2\right)I_x^2p^2\mathrm{v}^2+d^2\left(m\mathrm{v}^2C_M\left(M,\alpha^2\right)+p^2I_xC_{mag}(M)\right)\right]}; \quad (18)$$

and the equation for the square of the modulus of the angle of attack

$$\alpha^2=\alpha_1^2+\alpha_2^2+\alpha_3^2=\frac{4m^2p^2g^2I_x^2\left(\mathrm{v}_1^2+\mathrm{v}_3^2\right)}{\rho S\mathrm{v}^2\left[C_L^2\left(M,\alpha^2\right)I_x^2p^2\mathrm{v}^2+d^2\left(m\mathrm{v}^2C_M^2\left(M,\alpha^2\right)+p^2I_xC_{mag}(M)\right)\right]}. \quad (19)$$

The system of equations (13)-(18) is an explicit system of differential-algebraic equations, which takes into account all possible variants of the dependence of aerodynamic coefficients on the square of angle of attack modulus. Numerous methods for solving differential-algebraic equations are developed in more depth than for solving implicit differential equations. Therefore, with the availability of an appropriate set of linear and nonlinear aerodynamic coefficients, the system of equations (13)-(15) with equations (19) and (2) allows us to calculate the main parameters of the projectile flight: trajectories, projectile rotation speed, and the square angle of attack modulus, which are necessary for the design, modeling, and practical use of its ballistic characteristics.

3. Reduction of the system of differential equations of the MMPM to a system of algebraic expressions for identifying the aerodynamic coefficients of the projectile. Let us solve algebraically the system of equations (13)-(15), taking into account equations (19) and (2), relative to C_D, C_L, C_{mag}, C_{spin}, C_M, and obtain the following dependences of the aerodynamic coefficients:

$$C_D\left(M,\alpha^2\right)=\frac{2m\left[\mathrm{v}_1\frac{d}{dt}u_1+\mathrm{v}_2\left(\frac{d}{dt}u_2+g\right)+\mathrm{v}_3\frac{d}{dt}u_3\right]}{\rho S\mathrm{v}^3}; \quad (20)$$

$$\frac{C_L\left(M,\alpha^2\right)}{C_M\left(M,\alpha^2\right)}=-\frac{dmg\,\mathrm{v}^2\left[\mathrm{v}_1\frac{d}{dt}u_3-\mathrm{v}_3\frac{d}{dt}u_1\right]}{I_xp\left[\left(\mathrm{v}_2\frac{d}{dt}u_1-\mathrm{v}_1\frac{d}{dt}u_2\right)^2+\left(\mathrm{v}_3\frac{d}{dt}u_1-\mathrm{v}_1\frac{d}{dt}u_3\right)^2+\left(\mathrm{v}_3\frac{d}{dt}u_2-\mathrm{v}_2\frac{d}{dt}u_3\right)^2\right]}; \quad (21)$$

$$\frac{C_{mag}(M)}{C_M\left(M,\alpha^2\right)}=\frac{m\,\mathrm{v}^2}{I_x p^2}\left(\frac{g\left[\mathrm{v}_1\mathrm{v}_2\frac{d}{dt}u_1-\left(\mathrm{v}_1^2+\mathrm{v}_3^2\right)\frac{d}{dt}u_2+\mathrm{v}_2\mathrm{v}_3\frac{d}{dt}u_3\right]}{\left(\mathrm{v}_2\frac{d}{dt}u_1-\mathrm{v}_1\frac{d}{dt}u_2\right)^2+\left(\mathrm{v}_3\frac{d}{dt}u_1-\mathrm{v}_1\frac{d}{dt}u_3\right)^2+\left(\mathrm{v}_3\frac{d}{dt}u_2-\mathrm{v}_2\frac{d}{dt}u_3\right)^2}-1\right); \quad (22)$$

$$\alpha^2 C_M^2\left(M,\alpha^2\right)=\frac{4I_x^2p^2\left[\left(\mathrm{v}_2\frac{d}{dt}u_1-\mathrm{v}_1\frac{d}{dt}u_2\right)^2+\left(\mathrm{v}_3\frac{d}{dt}u_1-\mathrm{v}_1\frac{d}{dt}u_3\right)^2+\left(\mathrm{v}_3\frac{d}{dt}u_2-\mathrm{v}_2\frac{d}{dt}u_3\right)^2\right]}{\rho^2S^2d^2\,\mathrm{v}^8}. \quad (23)$$

Solving the equation of rotation of the projectile around its polar axis (2) with respect to the aerodynamic coefficient C_{spin}, we obtain

$$C_{spin}(M)=\frac{8I_x\frac{d}{dt}p}{\pi\rho d^4\,\mathrm{v}\,p}, \quad (24)$$

It is worth noting that the obtained system of equations (20)-(24) is equivalent to the system of equations (13)-(15), (19) and (2). In addition, we note an important circumstance, the left-hand sides of expressions (21)-(23) indicate that in MPMTM one should “painlessly” introduce the lift force and Magnus force normalized by the coefficient C_M, and also replace the square of the nutation angle modulus by $\alpha^2C_M^2$. This fact was noted earlier in works [17, 18].

Let us pay special attention to the fact that the right-hand sides of expressions (20)-(24) depend, with the exception of the projectile rotation speed *p*, only on the trajectory parameters, which can be measured experimentally, for example, using a radar. Thus, recording the flight coordinates of the projectile $x(t), y(t), z(t)$, it is possible to obtain from them (for example, by approximating them by smooth functions) the readings of the projectile flight speed values $\mathrm{v}_1(t), \mathrm{v}_2(t), \mathrm{v}_3(t)$, and similarly, the components of its acceleration. Using meteorological data, taking into account the coordinates of the projectile's flight, it is easy to calculate the air density $\rho(t)$, as well as the Mach number $M(t)$ on the trajectory. As for the projectile rotation speed $p(t)$, it can also be restored from the trajectory data listed above [21].

Using expressions (20)-(24) and substituting into them the readings of the values of the parameters listed above, we obtain the readings of the desired aerodynamic coefficients and the squares of the modulus of the projectile's angle of nutation.

Conclusion

The article presents procedures for equivalent transformations of a system of equations generalized relative to the standardized STANAG 4355 MMPM. An equivalent initial differential-algebraic system of equations was obtained, which, given the appropriate set of linear and nonlinear aerodynamic coefficients, allows calculating the main parameters of the projectile's flight with the same accuracy, but with lower computational resource consumption. By solving the algebraically obtained system with respect to the aerodynamic coefficients included in it, a system of fairly simple, accurate explicit expressions for their identification is obtained.

A system of accurate explicit expressions for identifying linear and nonlinear aerodynamic coefficients in further research will allow analyzing the performance of the identification method, assessing the errors of the method, and significantly accelerating the process of identifying both linear and nonlinear aerodynamic coefficients and calculating nutation angles from trajectory data.

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ІДЕНТИФІКАЦІЯ НЕЛІНЕЙНИХ АЕРОДИНАМІЧНИХ КОЕФІЦІЄНТІВ СНАРЯДА НА ОСНОВІ МОДИФІКОВАНОЇ МОДЕЛІ МАТЕРІАЛЬНОЇ ТОЧКИ

Грабчак В.І., Косовцов А.Ю., Грабчак В.В.

Для підвищення ефективності розрахунку траєкторій польоту снаряда на основі математичних моделей, що описують просторовий рух снаряда в повітрі, гостро стоїть питання визначення індивідуальних аеродинамічних коефіцієнтів снаряда із заданою точністю. В основі побудови сучасних математичних моделей польоту снаряда покладений наближений підхід, який отримав назву малокутового наближення. За таким підходом аеродинамічні коефіцієнти розкладають в ряд Тейлора за кутом нутації і утримуються лише лінійні члени розкладу, що дозволяє суттєво спростити математичні моделі польоту снаряда, але значно погіршує точність розрахунку траєкторій його польоту. Найбільш придатною для визначення аеродинамічних коефіцієнтів снаряда є модифікована модель матеріальної точки, як математична модель польоту снаряда (STANAG 4355 (Edition 3)). В статті представлені процедури перетворення модифікованої моделі матеріальної точки до системи диференціально-алгебраїчних рівнянь наданих у явному вигляді, яка, за наявності відповідного набору лінійних і нелінійних аеродинамічних коефіцієнтів, дозволяє обчислювати основні параметри польоту снаряда з меншими обчислювальними ресурсами. Отримано аналітичні вирази для ідентифікації аеродинамічних коефіцієнтів сили лобового опору, піднімальної сили, сили Магнуса, гасіння швидкості обертання снаряда, його перекидального моменту, а також квадрата модуля кута нутації снаряда. Показано, що за умови відомої функції зміни величини кутової швидкості власного обертання снаряда, отримані аналітичні вирази функціонально залежить виключно від параметрів, що отримуються за даними зовнішньо-траєкторних вимірювань (координат польоту снаряда та їх похідних).

Ключові слова: *снаряд, аеродинамічна сила, лінійні та нелінійні аеродинамічні коефіцієнти, математична модель, модифікована модель, ідентифікація, параметри польоту, нутація.*