РОЗРОБЛЕННЯ ТА МОДЕРНІЗАЦІЯ ОВТ

DEVELOPMENT AND MODERNIZATION OF WEAPONS AND MILITARY EQUIPMENT

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DETERMINING THE SPEED OF A PROJECTILE AT THE MOMENT OF ITS TRANSITION FROM SUBSONIC TO TRANSONIC SPEED

The authors have developed a method that takes into account that the magnitude of air drag force depends on the type of speed. That is, it has a different functional dependence when the projectile moves at supersonic, subsonic, or transonic speeds. The moment of transition of the projectile's speed from supersonic to subsonic is determined by the condition that the speed of the projectile becomes equal to the speed of sound in air. The latter depends on the air temperature at the point where the projectile is located. As the altitude of the projectile increases, the speed of sound in the air decreases. The condition that determines the speed of the projectile, which separates the movement of the projectile at subsonic speed from the movement at transonic speed, is not given in the literature. Based on the analysis of experimental studies aimed at establishing the dependence of the drag coefficient on the Mach number, the authors proposed a condition that allows determining the moment of change in the projectile's velocity from subsonic to transonic. It has been established that the speed value that distinguishes a subsonic projectile from a transonic V_{trsub} *depends on the mass and initial velocity of the projectile, air temperature, and atmospheric pressure. Numerical values of this magnitude* V_{trsub} *were obtained for Projectiles, HE, M795 and M795M; Fuze, PD, M739A1 fired from an M777A2 howitzer with a 3H charge. It has been determined that changes in the mass and initial velocity of the projectile have a negligible effect on the velocity* V_{trsub}*. However, changes in air temperature and atmospheric pressure have a significant effect on the velocity* V_{trsub}*. As the air temperature increases, the velocity* V_{trsub} *increases, and it also increases as the atmospheric pressure decreases.*

Keywords: *external ballistics, air drag force, subsonic projectile velocity, transonic velocity.*

Statement of the problem

When a projectile moves through the air, its speed can be supersonic, subsonic, or transonic. Experimental and theoretical studies have established that the magnitude of the air drag force on a projectile's motion depends significantly on the type of its speed. The graphical results of experimental studies conducted to establish the dependence of the drag coefficient on the Mach number for different types of projectiles are given in [1, 2].

Fig. 1 shows the approximate dependence of the drag coefficient CD on the Mach number. The dependence of the drag coefficient on the projectile's velocity is described by curve DB for transonic speeds, curve BC for subsonic speeds, and curve CL for supersonic speeds. Thus, the functional dependencies of the frontal air resistance force on the projectile velocity significantly depend on the type of velocity. Therefore, when studying the motion of a projectile, it is necessary to distinguish between the motions of a projectile at supersonic, subsonic, and transonic velocities.

The speed magnitude that separates the movement of a projectile from supersonic to subsonic speed depends on the air temperature at the point where the projectile is located and is determined using the formula

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$$V_s(z)=\sqrt{\frac{k_a R_{un}}{\mu_a}(TK-0{,}006328\,z)}\,, \qquad (1)$$

where V_s – is the speed of sound in air, $k_a=1{,}4$ – is the adiabatic index of air, $\mu_a=28{,}96$ kg/kmol – is the conditional molar mass of air, $R_{un}=8314$ J/kmol·K – is the universal gas constant, TK – is the absolute temperature of air at the weapon location, z – is the height of the projectile above the weapon location.

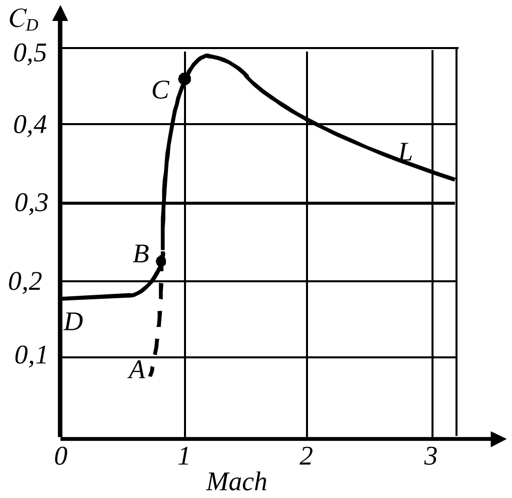

Fig. 1. **Dependence of the drag coefficient C_D on the Mach number**

The functional dependence for determining the velocity of a projectile which distinguishes subsonic velocity from transonic velocity is not explicitly stated in the literature.

Therefore, the problem of determining this velocity arises.

Analysis of research and publications

A significant number of scientific articles and experimental studies are devoted to the study of external ballistics problems. A summary of these results is presented in monographs [1-3]. In these monographs, scientific works [4-9], and other articles, the force of frontal air resistance to the motion of a projectile is described by dependencies

$$R=\frac{\rho V^2}{2}\frac{\pi d^2}{4} i c_x\left(\frac{V}{V_s}\right), \qquad (2)$$

or

$$R=\frac{\rho V^2}{2}\frac{\pi d^2}{4} C_D, \qquad (3)$$

where R – is the drag force of air, ρ – is the density of air, V – is the velocity of the projectile, d – is the caliber of the projectile, i – is the shape coefficient of the projectile, $c_x\left(\frac{V}{V_s}\right)$ – is the reference drag function, C_D – is the drag coefficient. The magnitude $\frac{\rho V^2}{2}$ is called dynamic pressure.

To reduce discrepancies between the results of field and theoretical studies of projectile dynamics, specific values of drag coefficients C_D or projectile shapes i are selected. Formulas (2) and (3) do not directly take into account the type of projectile velocity to determine the magnitude of the air drag force on the projectile's motion. That is, formulas (2) and (3) do not fully reflect the physical processes that occur during the flight of a projectile in the air. However, they allow, with a certain degree of accuracy, to determine the kinematic parameters of the projectile's movement in the air. Further scientific research is needed to gain a better understanding of the physical processes that occur during the flight of a projectile in the air.

Formulation of the article's objective

The speed of a projectile in flight, depending on its initial velocity, can be supersonic, subsonic, or transonic. The nature of the change in the magnitude of the air resistance force acting on a moving projectile depends significantly on the type of velocity. There is a need to determine the velocities that separate supersonic from subsonic and subsonic from transonic velocities. In the first case, the speed is determined using formula (1). How to determine the speed of a projectile that separates subsonic speed from transonic speed is not specified in the literature.

Therefore, the purpose of this article is to determine the magnitude of this velocity and to study the effects of the projectile's mass, its initial velocity, air temperature, atmospheric pressure, wind speed, and other factors on its magnitude.

Presentation of the main material

Unlike the widely known formulas (2) and (3), which determine the magnitude of the frontal air resistance force on a moving projectile, articles [10, 11] formulate another mathematical model for determining this force. The model takes into account the fact that the magnitude of the drag force significantly depends on the type of velocity. Therefore, the determination of the magnitude of the drag force of air on a moving projectile is described by the functional dependence

$$R(t)=c_x\cdot\rho_a\cdot s_x\cdot(V(t))^{2+\gamma_i}\left(\frac{V(t)}{V_s(t)}\right)^{\beta_i} \qquad (4)$$

and the values of its coefficients $\gamma_i\,(i=1,2,3)$ and $\beta_i\,(i=1,2,3)$ are different at supersonic $(i=1)$, subsonic $(i=2)$ and transonic $(i=3)$ speeds.

In formula (4), the following symbols are used: c_x – coefficient that takes into account the aerodynamics of the projectile shape when air flows around it longitudinally, and proportionality; ρ_a – air density; s_x – maximum cross-sectional area of the projectile.

In the absence of wind, the functional dependence of the frontal air resistance force on the projectile velocity will be as follows [10, 11]

$$R(t)=\frac{c_x s_x \mu_a \cdot 101325}{R_{un}}\left(1-\frac{6{,}5(z+zp)}{288000}\right)^{5{,}255}\times$$
$$\times\frac{(V(t))^{2+\gamma_i+\beta_i}}{\left(\frac{k_a R_{un}}{\mu_a}\right)^{0{,}5\beta_i}(TK-0{,}006328\;z(t))^{1+0{,}5\beta_i}}. \quad (5)$$

Variable zp characterizes the atmospheric pressure at the location of the weapon. For example, if the atmospheric pressure is 750 mm Hg, then $zp=111{,}54$ m and if it is 760 mm Hg, then – $zp=0$.

We place the origin of the coordinate system $Oxyz$ at the location of the weapon. The axis Ox is directed toward the target and lies in the horizontal plane of the weapon, the axis Oz – is directed vertically upward, and the axis Oy is perpendicular to the plane Oxz, forming a right-handed coordinate system.

Let us determine the effects of the projectile's mass, its initial velocity, air temperature, and atmospheric pressure on the velocity V_{trsub}, that distinguishes the subsonic velocity of the projectile from the transonic velocity.

Example. Let's consider the motion of Projectiles, HE, M795 and M795M; Fuze, PD, M739A1 fired from an M777A2 howitzer with a 3H charge, which provides the initial velocity of the projectile $V_0=547$ m/s.

The motion of the projectile under the action of air drag, projectile weight, and Coriolis force was considered.

During the research, the following values were used: $m=46{,}948$ kg, $s_x=\pi\cdot 0{,}078^2$ m^2, $c_x=0{,}35$, $t=15^\circ C$ or $TK=288\;K$, $p_{atm}=760$ mm Hg, wind speed – $V_W=0$ m/s.

At aiming angles within the range $0\,mil<\theta_0\le 125{,}1\,mil$ the projectile will only move at supersonic speed, and the values of the coefficients in formula (5) are as follows: $\gamma_1=-0{,}107512$ and $\beta_1=-0{,}044864$.

If the aiming angle changes within $128{,}4\;mil\le\theta_0\le 298{,}0\;mil$, then the projectile's velocity will initially be supersonic and then subsonic. The condition for the completion of the projectile's supersonic velocity phase and the start of its subsonic velocity phase is the violation of the inequation

$$V(t)\ge V_s(t). \quad (6)$$

At the stage of movement with subsonic velocity, the values of the parameters in formula (5) are as follows: $\gamma_2=-0{,}196753$ and $\beta_2=2{,}090781$.

It should be noted that the speed of sound decreases with increasing altitude of the projectile.

At aiming angles within $303{,}2\,mil\le\theta_0\le 573{,}3\,mil$ during the flight of the projectile, three types of velocity are recorded. Initially, the projectile moves at supersonic velocity, and the moment of completion of this stage is described by condition (6). In the second stage, it moves at subsonic velocity, and then at transonic velocity.

In Fig. 1, the *CL* curve describes the dependence of the drag coefficient C_D on the Mach number if the projectile velocity is supersonic. The *BC* curve indicates the nature of the change in the drag coefficient C_D, if the projectile velocity is subsonic. The *DB* curve describes the change in the drag coefficient C_D with a change in the Mach number if the projectile is moving at transonic velocity. Point *B* is the point that separates the dependence of the drag coefficient C_D on the Mach number at subsonic speed from that at transonic speed. Analyzing the graph in Fig. 1, we can see that as the Mach number decreases, the drag coefficient C_D, according to the nature of the *SV* curve, does not continue to decrease sharply after point *B*. That is, a further decrease in the drag coefficient is not described by curve *AB*, but by curve *DB*. There is a tendency for the nature of the dependence of the drag coefficient on the Mach number to change. This gives reason to assert that point *B* separates the stage of projectile motion at subsonic speed from the stage of motion at transonic speed. Thus, the end of the stage of projectile motion at subsonic speed is the moment when the inequation condition is violated,

$$V(t)>V(t+\Delta t), \quad (7)$$

where $\Delta t>0$.

Let us analyze the influence of individual parameters on the velocity of a projectile at the moment of its transition from subsonic to transonic speeds.

Let us consider the influence of a change in the mass of the projectile, under otherwise standard conditions, on the velocity V_{trsub} that distinguishes the movement of a projectile at subsonic speed from transonic speed.

Table 1

Dependence of the projectile velocity at the moment of transition from subsonic to transonic speed on its mass

m, kg	V_0, m/s	θ_0, mil	V_{trsub}, m/s	t_k, sec
$2sq$ (45,950)	552,91	298,677	301,21	26,609
$3sq$ (46,449)	549,93	298,568	301,73	26,541
$4sq$ (46,948)	547,00	298,451	302,24	26,472
$5sq$ (47,447)	544,12	298,315	302,74	26,401
$6sq$ (47,946)	541,28	298,163	303,23	26,329

Table 1 shows the weight of the projectile in the first column. Due to changes in the weight of the projectile, its initial velocity will change with the same charge, and its magnitudes are shown in the second column. The third column shows the aiming angle at which the projectile completes its movement at subsonic speed when it reaches the level of the weapon. The fourth column shows the projectile's velocity V_{trsub} at the moment of transition from subsonic to transonic speeds. The duration of the projectile's movement is shown in the fifth column.

Analyzing the results presented in Table 1, we can conclude that increasing the mass of the projectile does not significantly increase the velocity V_{trsub}, that separates the projectile's movement from subsonic to transonic speeds.

Let us examine the effect of changing the initial velocity of the projectile, under otherwise standard conditions, on the velocity V_{trsub} at the moment of transition from subsonic to transonic speed.

Table 2

Dependence of the projectile velocity at the moment of transition from subsonic to transonic velocity on its initial velocity

V_0, m/s	θ_0, mil	V_{trsub}, m/s	t_k, sec
539,00	298,470	301,61	26,187
543,00	298,465	301,93	26,330
547,00	298,451	302,24	26,472
551,00	298,425	302,54	26,612
555,00	298,391	302,85	26,752

Analyzing the results presented in Table 2, it can be stated that increasing the initial velocity of the projectile does not significantly increase the velocity V_{trsub} that distinguishes the movement of the projectile at subsonic and transonic speeds.

Let us consider the effect of only the change in air temperature, under otherwise standard conditions, on the speed V_{trsub} at the moment of transition from subsonic to transonic speed.

Table 3

Dependence of the projectile velocity at the moment of transition from subsonic to transonic speed on the change in air temperature

$t^{\circ}C$	θ_0, mil	V_{trsub}, m/s	t_k, c
$-5^{\circ}C$	297,256	292,34	26,102
$5^{\circ}C$	297,906	297,35	26,294
$15^{\circ}C$	298,451	302,24	26,472
$25^{\circ}C$	298,894	307,02	26,637
$35^{\circ}C$	299,245	311,69	26,791

Analyzing the results presented in Table 3, it can be stated that an increase in air temperature significantly increases the speed V_{trsub} that distinguishes subsonic projectile motion from transonic motion.

Changes in atmospheric pressure also affect the speed V_{trsub} that distinguishes subsonic projectile motion from transonic motion.

Table 4

Dependence of the projectile velocity at the moment of transition from subsonic to supersonic speed on changes in atmospheric pressure

p_{atm}, mm Hg	θ_0, mil	V_{trsub}, m/s	t_k, sec
760	298,451	302,24	26,472
740	298,171	304,09	26,560
720	297,848	305,99	26,647
700	297,476	307,95	26,733
680	297,047	309,96	26,817

Analyzing the results presented in Table 4, it can be stated that a decrease in atmospheric pressure significantly increases the speed V_{trsub} that distinguishes the movement of a projectile at subsonic speed from transonic speed.

It can be argued that accompanying and side winds will also affect the speed V_{trsub} that distinguishes the movement of a projectile traveling at subsonic speed from transonic speed.

When the aiming angle increases $298{,}0 mil < \theta_0$, a gradual change in the projectile's velocity will be observed during its flight. Initially, it will move at supersonic speed, and the end of this stage is characterized by a violation of condition (6). In the second stage, the projectile moves at subsonic speed. Condition (7) allows us to determine the moment when the projectile completes its subsonic motion and begins to move at transonic speed. In the third stage, the projectile moves at transonic speed, and in formula (4), the values of the parameters $\gamma_3 = -0{,}253595$ and $\beta_3 = 0{,}777779$.

During mortar firing, certain peculiarities of the projectile's movement will be observed, which will be discussed in a separate article.

Conclusions

During the flight of Projectiles, HE, M795 and M795M; Fuze, PD, M739A1 fired from the M777A2 howitzer and charged with 3H, at aiming angles $\theta_0 < 746{,}6 mil$, the projectile will gradually move at supersonic, subsonic, and transonic speeds. The transition of the projectile from supersonic to subsonic speed is described by inequation (6), and the transition from subsonic to transonic speed is determined by condition (7).

The speed magnitude V_{trsub} that separates the movement of a projectile from subsonic to transonic speeds depends on the mass of the projectile, its initial speed, air temperature, and atmospheric pressure. It can be argued that the speed magnitude V_{trsub} is influenced by both headwinds and crosswinds. Further research will be devoted to this issue.

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ВИЗНАЧЕННЯ ВЕЛИЧИНИ ШВИДКОСТІ СНАРЯДА В МОМЕНТ ПЕРЕХОДУ ЙОГО РУХУ ВІД ПІДЗВУКОВОЇ ДО ДОЗВУКОВОЇ ШВИДКОСТЕЙ

Л.Д. Величко, М.І. Войтович, М.І. Сорокатий

Авторами розроблена методика, яка враховує, що величина сили лобового опору повітря залежить від типу швидкості. Тобто має різну функціональну залежність при русі снаряда з надзвуковою, підзвуковою чи дозвуковою швидкостями. Момент переходу швидкості снаряда з надзвукової на підзвукову визначається з умови, що швидкість снаряда стає рівною швидкості звуку в повітрі. Остання залежить від температури повітря в точці перебування снаряда. Зі збільшенням висоти руху снаряда величина швидкості звуку в повітрі зменшується. Умову, з якої визначається величина швидкості снаряда, що відокремлює рух снаряда з підзвуковою швидкістю від руху з дозвуковою, в літературі не наведено. На основі аналізу експериментальних досліджень, скерованих на встановлення залежності коефіцієнта сили опору від числа Маха, автори запропонували умову, яка дозволяє визначати момент зміни швидкості снаряда з підзвукової на дозвукову. Встановлено, що величина швидкості, яка розмежовує рух снаряда з підзвуковою швидкістю від дозвукової V_{trsub}*, залежить від маси та початкової швидкості снаряда, температури повітря і атмосферного тиску. Отримано числові значення цієї величини* V_{trsub} *для Projectiles, HE, M795 і M795M; Fuze, PD, M739A1, випущеного з гаубиці M777A2 із зарядом 3H. Визначено, що зміна маси і початкової швидкості снаряда незначно впливає на величину швидкості* V_{trsub}*. Проте зміна температури повітря та атмосферного тиску суттєво впливають на величину швидкості* V_{trsub}*. При збільшенні температури повітря величина швидкості* V_{trsub} *зростає і вона теж зростає при зменшенні атмосферного тиску.*

Ключові слова: *зовнішня балістика, сила лобового опору повітря, підзвукова швидкість снаряда, дозвукова швидкість.*